

Cosmological Constraints on Unparticles as Continuous Mass Particles

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Abstract

We study the cosmological constraints on unparticle interactions and the temperature of the Universe for the case where unparticle states can be modelled as continuous mass particles with lifetime $\gtrsim 1\text{s}$. By considering thermal background quark decay to continuous mass scalars via a scalar operator of dimension d_U , we show that the condition that the Universe is not dominated by scalars at nucleosynthesis imposes a lower bound on the scale of the interaction of the unparticle sector, with $M_U \gtrsim 20 - 2600 \text{ TeV}$ for $\Lambda_U \gtrsim 1 \text{ TeV}$, $1.1 \leq d_U \leq 2.0$ and $2 \leq d_{BZ} \leq 4$. The existence of a long-lived scalar sector also imposes an upper bound on the temperature of the Universe during radiation-domination, which can be as low as a TeV for M_U close to its lower bound.

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1 Introduction

Recently there has been much interest in the possible existence of a conformally-invariant hidden sector consisting of 'unparticles' [1, 2]¹; see [4-11]. In this picture, the conformally-invariant unparticle sector is generated non-perturbatively from a Banks-Zaks (BZ) sector consisting of non-Abelian gauge fields and vector pairs of massless fermions [12], which are assumed to interact very weakly with the SM sector via operators suppressed by a mass scale M_U . The BZ sector undergoes dimensional transmutation to the unparticle phase, corresponding to a strongly-coupled conformal field theory (CFT), below an infra-red fixed point energy Λ_U . This unparticle phase has no conventional particle description but interacts with the SM via unparticle operators of mass dimension d_U , which create unparticle states. The remarkable characteristic of unparticles is that the phase space in decay processes to unparticle stuff is the same as the phase space for decay to d_U massless particles, where d_U can be non-integral. This could provide a distinctive signal for unparticle production in collider experiments in the case where the unparticle description is valid at future collider energies, corresponding to $\Lambda_U \gtrsim 1$ TeV.

The couplings of the unparticle operators and the value of d_U will depend on the underlying BZ operators and the infra-red dynamics of dimensional transmutation. Since these are model-dependent, most studies of unparticle physics have introduced several possible unparticle operators with unknown coefficients and investigated their consequences. Under reasonable assumptions regarding the unknowns, the mass scale M_U can then be constrained by the requirement that the unparticle sector does not conflict with present phenomenological [4], astrophysical [5], long-range force [6] and cosmological [8] observations.

Here we consider the effect of an unparticle sector on the cosmology of the SM and the resulting constraints on unparticle interactions and the temperature of the Universe. This requires a model for the unparticle states and their interactions with the SM. In [9], the production of unparticle states by scattering of SM particles was

¹Similar ideas were proposed earlier in [3].

modelled by scalar particles with a continuous mass. (See also [10, 11].) Although this picture does not fully describe the strongly-coupled unparticle CFT, we will consider it as a starting point from which to study the cosmology of the unparticle states. Therefore in the following we will study the cosmology of continuous mass scalars as motivated by unparticles. Our focus will be the decay and annihilation rates of thermal background SM quarks to unparticle-based continuous mass scalars. For clarity, we focus on a specific interaction between SM quarks and a scalar unparticle operator, leaving a more general operator analysis for future work.

Our analysis is based on the assumption that the cosmology of continuous mass scalars will approximate that of unparticles. It should be emphasized that this is a non-trivial assumption, which can only be validated with a deeper study of unparticle cosmology based on a full CFT analysis. Continuous mass scalars imply the existence of asymptotic states, which do not formally exist in a CFT. Therefore continuous mass scalars cannot fully describe the physics of unparticles. The degree to which they correctly describe unparticle cosmology is therefore dependent on the degree to which continuous mass scalars accurately describe the production of unparticle states in particle physics processes and the stability of the unparticle energy density.

The scale-invariance of the unparticles must be broken in order that they are not overproduced in astrophysical processes, which requires that there is a mass gap larger than 30 MeV [5]. The unparticle description is valid for unparticle production occurring at energy scales larger than the mass gap, otherwise a standard particle cosmology applies. We therefore assume that the mass gap is sufficiently small that there is a range of temperature between the mass gap and Λ_U for which unparticle states are produced.

The article is organised as follows. In Section 2 we review unparticles and their interpretation in terms of deconstructing scalars. We also relate these scalars to a physical interpretation of the unparticle states. In Section 3 we calculate the decay and pair annihilation rates of thermal SM quarks to unparticle-based continuous mass scalars and derive bounds from requiring that the scalar energy density does not disturb nucleosynthesis. In Section 4 we discuss our conclusions.

2 The Model

The unparticle model [1, 2] is based on an interaction between Banks-Zaks (BZ) fields and SM fields mediated by exchange of heavy particles of mass M_U

$$\frac{1}{M_U^k} O_{SM} O_{BZ} . \quad (1)$$

O_{BZ} is an operator with mass dimension d_{BZ} made out of BZ fields. At energy scales less than Λ_U the BZ sector undergoes dimensional transmutation and a scale-invariant unparticle sector, corresponding to a strongly self-coupled CFT, is formed. The BZ operators match onto unparticle operators and Eq. (1) becomes

$$\frac{C_U \Lambda_U^{d_{BZ}-d_U}}{M_U^k} O_{SM} O_U , \quad (2)$$

where d_U is the scaling dimension of the unparticle operators O_U . The value of C_U and d_U will be determined by the infra-red dynamics of the Banks-Zaks fields and the form of O_{BZ} . These will be considered as free parameters in the following. To study how the unparticle sector influences the cosmology of the SM, we focus on a specific interaction between SM quarks and a scalar unparticle operator O_U [1],

$$\frac{i\lambda}{\Lambda_U^{d_U}} \bar{q} \gamma_\mu (1 - \gamma_5) q' \partial^\mu O_U + h.c. . \quad (3)$$

Here $\lambda \equiv C_U (\Lambda_U / M_U)^{d_{BZ}}$ is a dimensionless coupling.

A useful conceptual framework for understanding unparticle production was presented in [9]. It was shown that by breaking scale-invariance in a controlled way, the continuous energy spectrum of the unparticle sector can be replaced by a discrete tower of deconstructing scalar particles ϕ_n of mass M_n , with mass spacing controlled by a mass parameter Δ such that $M_n^2 = n\Delta^2$. In this case the unparticle operator O_U is replaced by a sum over canonically normalized deconstructing scalar fields ϕ_n ,

$$\sum_{n=1}^{\infty} \frac{i\lambda F_n}{\Lambda_U^{d_U}} \bar{q} \gamma_\mu (1 - \gamma_5) q' \partial^\mu \phi_n + h.c. , \quad (4)$$

where

$$F_n^2 = \frac{A_{d_U}}{2\pi} \Delta^2 (M_n^2)^{d_U-2}$$

and

$$A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + 1/2)}{\Gamma(d_U - 1)\Gamma(2d_U)}.$$

A_{d_U} is a conventionally chosen phase space factor for unparticles [1]; only the combination $C_U A_{d_U}^2$ appears in physical processes. In the limit $\Delta \rightarrow 0$, the sum over decays $q' \rightarrow q + \phi_n$ for all kinematically allowed values of n produces an expression for $d\Gamma/dE_q$ in agreement with the direct unparticle calculation of $q' \rightarrow q + U$ based on O_U [9]. The individual $q + \phi_n$ final states of energy $E_q = (m_{q'}^2 - M_n^2)/2m_{q'}$ merge into a continuum decay rate as a function of E_q as $\Delta \rightarrow 0$, reproducing the unparticle result.

The scalar particle deconstruction of the unparticle propagator does not completely describe the unparticle CFT. However, in cosmology we are primarily interested in the scattering between SM particles and unparticle states. In this case the continuous mass scalars may be a useful approximation to the unparticle states. If we consider the unparticle sector to be the strongly-coupled limit of a Banks-Zaks sector, then the states will correspond to ‘bound states’ of Banks-Zaks fermions and gauge fields. Since the theory is scale-invariant in the strongly-coupled limit, there can be no preferred mass associated with the bound states. These objects will therefore have all kinematically possible masses as a result of the scale-invariance of the underlying CFT. Therefore the unparticle states may be expected to behave approximately as particle-like states with a continuous mass parameter. This picture cannot be exactly correct however, as CFTs do not have asymptotic states. In what follows we are therefore assuming that continuous mass particles are a sufficiently good approximation to the unparticle states that they can be used to understand the production and stability of an unparticle energy in a cosmological setting. Deviations from the unparticle picture may also be expected due to the finite value of Λ_U relative to the interaction energy in scattering processes, which breaks true scale-invariance.

3 Cosmological constraints from quark decay and annihilation to continuous mass scalars

We will consider the couplings Eq. (3) and Eq. (4) in the context of cosmology. In particular, we will calculate the decay and annihilation rates of thermal background quarks to continuous mass scalars to obtain the conditions under which these processes can produce a significant energy density and the resulting constraints on cosmology.

A basic property of the continuous mass particle sector from unparticle deconstruction is that the particles are stable [9]. This can be understood by considering the deconstructing scalars in the limit $\Delta \rightarrow 0$. In this limit the couplings of the scalars, which are proportional to $F_n \propto \Delta^{d_U-1}$, tend to zero if $d_U > 1$. The reason the decay and scattering rates of SM particles to continuous mass scalars is finite in the $\Delta \rightarrow 0$ limit is that the decrease in the rate to a scalar particle final state ϕ_n is compensated for by an increase in the number of kinematically allowed scalar final states $\approx E^2/\Delta^2$, where E is the energy the decaying or annihilating quark. (We will demonstrate this explicitly for the case of pair annihilation of SM quarks to continuous mass scalars.) However, if we consider the decay and scattering of continuous mass scalars to SM particles, then as $\Delta \rightarrow 0$ there is no compensating increase in the number of SM particles. So the rates of all processes which could result in the decay of the scalar particle energy density to SM particles tend to zero in the continuous mass limit $\Delta \rightarrow 0$ [9]. Since the cosmological constraints on unparticles follow from their effect on nucleosynthesis, unstable unparticles will be subject to the same constraints if their lifetime is greater than $O(1)$ s.

Since an unparticle sector is scale-invariant, the unparticle density evolves with scale factor as $\rho_U \propto a^{-4}$ [8]. We will assume the continuous mass scalar density also evolves as a^{-4} . (If it evolves as a^{-n} with $n < 4$ then the cosmological constraints become stronger.) If the scalar density decouples from the SM density at $T_{DEC} \gg T_{BBN} \approx 0.1$ MeV, where T_{BBN} is the temperature at Big-Bang Nucleosynthesis (BBN), then a dilution of the continuous mass scalar density relative to the radiation density is possible, due to subsequent photon heating by annihilation of massive

SM particles. However, the maximum dilution relative to photons is only by a factor $(g_*(T_{BBN})/g_*(T_{DEC}))^{4/3} (g(T_{DEC})/g(T_{BBN}))$, where g and g_* are the effective number of relativistic degrees of freedom associated with energy and entropy respectively, since the radiation energy density is proportional to $g(T)T^4$ while the continuous mass scalar density redshifts as a^{-4} with $g_*(T)a^3T^3 = \text{constant}$. The maximum dilution is obtained if decoupling occurs at a higher temperature than the electroweak phase transition, in which case $g(T_{DEC}) = g_*(T_{DEC}) = 106.75$. With $g(T_{BBN}) = 3.36$ and $g_*(T_{BBN}) = 3.90$ for photons plus neutrinos, this gives dilution by at most a factor 0.39. This is not enough to dilute the continuous mass scalar density sufficiently unless $\rho_U < 0.15\rho_{SM}$ at T_{DEC} , since successful nucleosynthesis requires that any additional energy density be less than 6% of the total at T_{BBN} [13]. Therefore in order that the successful light element abundance predictions of BBN are unaffected, SM quark decay or pair annihilation to continuous mass scalars should not produce a large continuous mass scalar density.

3.1 Quark decay to continuous mass scalars

The interaction Eq. (4) implies a differential decay rate to deconstructing scalars (equal to the decay rate to unparticles) $q' \rightarrow q + U$ given by [1, 9]

$$\frac{d\Gamma}{dE_q} = \frac{|\lambda|^2 m_{q'}^2 A_{d_U} E_q^2 (m_{q'}^2 - 2m_{q'} E_q)^{d_U-2}}{2\pi^2 \Lambda_u^{2d_U}}. \quad (5)$$

This is the decay rate in the q' quark rest frame. Integrating this for all energies up to $m_{q'}/2$ gives

$$\Gamma = \frac{|\lambda|^2}{8\pi^2} \frac{A_{d_U} m_{q'}^{2d_U+1}}{d_U(d_U^2 - 1)\Lambda_U^{2d_U}}. \quad (6)$$

In the limit $d_U \rightarrow 1$ the decay rate becomes infinite, therefore $d_U > 1$ must be imposed [1], in agreement with general unitarity arguments for a scalar unparticle operator. For thermal relativistic quarks of mean energy $E \approx 3T$, the decay rate Γ_d is obtained from the rest frame decay rate by dividing by the Lorentz factor $\gamma = 3T/m_{q'}$. Therefore

$$\Gamma_d \approx \frac{|\lambda|^2}{8\pi^2} \frac{A_{d_U}}{d_U(d_U^2 - 1)\Lambda_U^{2d_U}} \frac{m_{q'}^{2d_U+2}}{3T}. \quad (7)$$

At $T < T_{EW}$, where T_{EW} is the temperature of the electroweak phase transition, we will assume that the Higgs expectation value is given by the $T = 0$ value² $\langle H \rangle = v$. At $T > T_{EW}$, we expect the effective mass of the quarks to be given by the temperature, $m_{q'} \approx T$. Therefore

$$\Gamma_d \approx \frac{C_U^2 A_{d_U}}{24\pi^2} \frac{\Lambda_U^{2(d_{BZ}-d_U)}}{d_U(d_U^2-1)M_u^{2d_{BZ}}} \frac{m_{q'}^{2(d_U+1)}}{T} \quad ; \quad T < T_{EW} \quad (8)$$

and

$$\Gamma_d \approx \frac{C_U^2 A_{d_U}}{24\pi^2} \frac{\Lambda_U^{2(d_{BZ}-d_U)} T^{2d_U+1}}{d_U(d_U^2-1)M_u^{2d_{BZ}}} \quad ; \quad T > T_{EW} . \quad (9)$$

In order not to produce a large continuous mass scalar density we must then impose that $\Gamma_d < H$, where during SM radiation-domination $H = k_T T^2/M$ with $k_T = (\pi^2 g(T)/90)^{1/2}$ and $M = M_{Pl}/\sqrt{8\pi}$.

For $T < T_{EW}$, the strongest constraint comes from the case of t quark decay. In this case the condition $\Gamma_d < H$ is strongest at the lowest value of T for which there are relativistic thermal t-quarks, $T \approx m_t/3$, which implies that

$$M_U^{2d_{BZ}} \gtrsim \frac{9}{8\pi^2} \frac{k_1}{k_T} \frac{m_t^{2d_U-1} M \Lambda_U^{2(d_{BZ}-d_U)}}{d_U(d_U^2-1)} , \quad (10)$$

where $k_1 = C_U^2 A_{d_U}$. If this condition is satisfied, then for $m_t/3 \lesssim T < T_{EW}$ there is no significant decay of the SM radiation to continuous mass scalars. Once $T \lesssim m_t/3$, the next possible decay is to b quarks, but the condition for this not to occur, which replaces m_t by m_b in Eq. (10), is automatically satisfied if Eq. (10) is satisfied³. On the other hand, if Eq. (10) is not satisfied, then as T decreases from T_{EW} , at some $T \gtrsim m_t/3$ the decay rate becomes faster than H . The SM radiation then loses energy to continuous mass scalars, decreasing T and increasing Γ_d relative to H (which remains constant throughout since the total energy density is constant) until $T \lesssim m_t/3$. As a result, if T is larger than $m_t/3$, a large fraction of the SM radiation energy will

²The Higgs expectation value is temperature dependent, given by $\langle H \rangle = v(1 - T^2/T_{EW}^2)^{1/2}$, where $T_{EW} \approx 1.2m_h$ [14]. For simplicity in our estimates we will use the approximation that $\langle H \rangle = 0$ at $T < T_{EW}$ and $\langle H \rangle = v$ at $T > T_{EW}$.

³Decay to other light quarks is also possible, but these decays will be subdominant due to the quark mass factor in Eq. (10).

be transferred to the continuous mass scalar sector, resulting in a scalar-dominated Universe.

For $T > T_{EW}$, the condition $\Gamma_d < H$ requires that

$$T^{2d_U-1} \lesssim 24\pi^2 \frac{k_T}{k_1} \frac{d_U(d_U^2 - 1)M_u^{2d_{BZ}}}{\Lambda_U^{2(d_{BZ}-d_U)}M} . \quad (11)$$

If this condition is not satisfied, then SM radiation will rapidly decay to continuous mass scalars until T has decreased sufficiently that the condition $\Gamma_d < H$ is satisfied. As a result, a large fraction of the SM radiation energy will have transferred to the scalar sector when the decay rate becomes negligible. Therefore there is an upper bound on the temperature of the radiation-dominated universe in the presence of a sufficiently stable (lifetime $\gtrsim 1$ s) continuous mass scalar sector.

These conclusions are based on the idea that any process which results in energy transfer will always be much more rapid from the SM to the continuous mass scalar sector than the reverse process. In the deconstructing scalar picture the unparticle states are stable with respect to decay and annihilation via the interaction Eq. (3). However, it is conceivable that unparticle self-interactions, which will depend on the details of the underlying CFT, could result in processes transferring energy to the SM. Nevertheless, the reverse process from SM to continuous mass scalars will always be much more rapid due to the large number of scalar particle final states.

We note that if the unparticle CFT can come into thermal equilibrium then it may be possible to evade these constraints, if the central charge c of the CFT is small enough. The energy density is $\rho_U = \pi c T^4/6$. Therefore if the unparticle density decouples at $T \gtrsim M_W$, the density at nucleosynthesis will be small enough if $\pi c/6 < 0.15 \times \pi^2 g(T)/30$ with $g(T) = 106.75$. This requires $c < 10$. A considerable reduction in the number of effective degrees of freedom, from 100 for the underlying Banks-Zaks theory to less than 10 for the unparticle CFT, would be necessary for this to work.

3.2 Quark pair annihilation to continuous mass scalars

We next consider the annihilation of thermal quark pairs to continuous mass scalars, by calculating the scattering cross-section to deconstructing scalars in the $\Delta \rightarrow 0$ limit.

The CM annihilation cross-section $q + q \rightarrow \phi_m + \phi_n$ following from Eq. (4) is given by

$$\sigma_{mn} \approx \frac{g_m^2 g_n^2 E^2}{M_s^4} , \quad (12)$$

where E is the energy of the annihilating quarks,

$$g_n^2 = \frac{|\lambda|^2}{n^{(2-d_U)}}$$

and

$$M_s^2 = \frac{2\pi}{A_{d_U}} \left(\frac{\Lambda_U}{\Delta} \right)^{2d_U} \Delta^2 .$$

(In this we have written the coupling for each n in Eq. (4) as g_n/M_s .) The total scattering rate to deconstructing scalars is obtained by summing over the kinematically allowed final states. With quark energy E , the final state scalars have $n_{max} = E^2/\Delta^2$, therefore

$$\sigma_{TOT} = \sum_{n=1}^{n_{max}} \sum_{m=1}^{n_{max}} \sigma_{mn} . \quad (13)$$

For $n_{max} \gg 1$ we can replace the sums with integrals,

$$\sum_{n=1}^{n_{max}} g_n^2 \approx \int_1^{E^2/\Delta^2} g_n^2 dn \equiv \int_1^{E^2/\Delta^2} \frac{|\lambda|^2}{n^{(2-d_U)}} dn . \quad (14)$$

Therefore

$$\sum_{n=1}^{n_{max}} g_n^2 \approx \frac{|\lambda|^2}{(d_U - 1)} \left[\left(\frac{E}{\Delta} \right)^{2(d_U-1)} - 1 \right] . \quad (15)$$

Thus summing over m and n is Eq. (13) gives

$$\sigma_{TOT} \approx \frac{E^2}{M_s^4} \frac{|\lambda|^4}{(d_U - 1)^2} \left[\left(\frac{E}{\Delta} \right)^{2(d_U-1)} - 1 \right]^2 . \quad (16)$$

Replacing M_s^2 by its definition then gives

$$\sigma_{TOT} \approx \frac{|\lambda|^4 E^2}{(d_U - 1)^2} \left[\left(\frac{E}{\Delta} \right)^{2(d_U-1)} - 1 \right]^2 \frac{A_{d_U}^2 \Delta^{4(d_U-1)}}{(2\pi)^2 \Lambda_U^{4d_U}} . \quad (17)$$

In the case $d_U > 1$, in the continuous mass limit $\Delta \rightarrow 0$ the cross-section tends to a finite limit

$$\sigma_{TOT} \approx \frac{|\lambda|^4}{(d_U - 1)^2} \frac{A_{d_U}^2 E^{4d_U-2}}{(2\pi)^2 \Lambda_U^{4d_U}} . \quad (18)$$

In the case $d_U \leq 1$, $\sigma_{TOT} \rightarrow \infty$ as $\Delta \rightarrow 0$. Therefore, as in the case of the quark decay rate, the quark annihilation rate is singular as $d_U \rightarrow 1$.

For $d_U > 1$ the annihilation rate is then $\Gamma_{ann} = n_q \sigma_{TOT}$, where $n_q = 12T^3/\pi^2$ is the number density of thermal quarks q . Therefore, with $E \approx 3T$,

$$\Gamma_{ann} \approx \frac{1}{9\pi^4} \frac{C_U^4 A_{d_U}^2 (3T)^{4d_U+1}}{(d_U - 1)^2 \Lambda_U^{4d_U}}. \quad (19)$$

In most cases this rate is small compared with the quark decay rate. However, for d_U sufficiently close to 1, the annihilation rate ($\propto (d_U - 1)^{-2}$) can become larger than the decay rate Eq. (7) ($\propto (d_U - 1)^{-1}$).

3.3 Cosmological constraints on unparticle parameters from continuous mass scalar cosmology

We next consider the constraints on the unparticle parameters M_U and Λ_U as a function of d_U following from thermal quark decay to continuous mass scalars. The bounds derived above assume that the energy of the quarks is less than Λ_U , otherwise the decay and annihilation rates should be calculated for BZ particle final states. Since the most interesting case is where unparticles may be observed in future colliders, we will consider $\Lambda_U \gtrsim 1$ TeV in the following. The decay rate depends upon the product $k_1 \equiv C_U^2 A_{d_U}$, which is determined by the infra-red dynamics of the Banks-Zaks fields [1]. Therefore any conclusions about the cosmology of unparticles is dependent upon assumptions about this product, which should become better understood as unparticle physics develops. In the following we will assume that k_1 is not very large or small compared with 1.

The constraints will depend on the assumed values of the model parameters d_{BZ} , d_U and Λ_U . We first focus on the case where the BZ operator is a quark bilinear, so that $d_{BZ} = 3$, and calculate constraints for the cases $d_U = 1.1, 1.5$ and 2 when $\Lambda_U \gtrsim 1$ TeV.

The most important constraints come from Eq. (10) when $T < T_{EW}$. This gives lower bounds on M_U as a function of d_U

$$\mathbf{d_U = 1.1 :} \quad M_U \gtrsim 190 \, k_1^{1/6} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{19/30} \text{ TeV}$$

$$\begin{aligned}
\mathbf{d_U} = \mathbf{1.5} : \quad M_U &\gtrsim 110 \, k_1^{1/6} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{1/2} \text{ TeV} \\
\mathbf{d_U} = \mathbf{2.0} : \quad M_U &\gtrsim 70 \, k_1^{1/6} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{1/3} \text{ TeV} ,
\end{aligned} \tag{20}$$

where we have used $k_T = 3$. Thus if k_1 is not very large or small compared with 1 and $\Lambda_U \gtrsim 1 \text{ TeV}$, the mass scale of the $d_{BZ} = 3$ interaction between the BZ and SM fields must be greater than around 100 TeV for d_U between 1.1 and 2.0. These bounds hold if the radiation-dominated era has $T \gtrsim m_t/3$ at some time, otherwise weaker bounds based on lighter quark decays will apply.

Upper bounds on the temperature of the radiation-dominated era when $T > T_{EW}$ can be obtained from Eq. (11). However, these bounds stricly hold only if the energy of the decaying quark $E \approx 3T$ is less than Λ_U , since for larger temperatures and energies the decay will produce BZ particles rather than unparticles. For $T_{EW} < T \lesssim \Lambda_U/3$ we obtain:

$$\begin{aligned}
\mathbf{d_U} = \mathbf{1.1} : \quad T &\lesssim 10 \text{ TeV} \left(\frac{1}{k_1} \right)^{5/6} \left(\frac{M_U}{250 \text{ TeV}} \right)^5 \left(\frac{1 \text{ TeV}}{\Lambda_U} \right)^{19/6} \\
\mathbf{d_U} = \mathbf{1.5} : \quad T &\lesssim 2.6 \text{ TeV} \left(\frac{1}{k_1} \right)^{1/2} \left(\frac{M_U}{150 \text{ TeV}} \right)^3 \left(\frac{1 \text{ TeV}}{\Lambda_U} \right)^{3/2} \\
\mathbf{d_U} = \mathbf{2.0} : \quad T &\lesssim 1.3 \text{ TeV} \left(\frac{1}{k_1} \right)^{1/3} \left(\frac{M_U}{100 \text{ TeV}} \right)^2 \left(\frac{1 \text{ TeV}}{\Lambda_U} \right)^{2/3} .
\end{aligned} \tag{21}$$

The upper bounds on T are sensitive to M_U and the value of $3T$ can easily be above Λ_U . In this case the decay should be calculated to BZ sector fields using the interaction Eq. (1). For $d_{BZ} = 3$ and O_{BZ} corresponding to a bilinear of BZ fermions $\bar{\Psi}\Psi$, the interaction with the SM quarks becomes

$$\frac{1}{M_U^k} O_{SM} O_{BZ} \rightarrow \frac{\partial_\mu (\bar{q} \gamma_\mu (1 - \gamma_5) q') \bar{\Psi} \Psi}{M_U^3} . \tag{22}$$

The decay rate for $q' \rightarrow q \bar{\Psi} \Psi$ is then

$$\Gamma_d \approx \frac{1}{(8\pi)^3} \frac{T^7}{M_U^6} \tag{23}$$

The condition $\Gamma_d < H$ then implies that

$$T \lesssim 1.8 \left(\frac{M_U}{100 \text{ TeV}} \right)^{6/5} \text{ TeV} . \tag{24}$$

If this bound is not satisfied, energy will flow to the BZ sector until thermal equilibrium is established with the BZ sector, resulting in a large BZ sector energy density. For example, for the case of the original Banks-Zaks theory with an $SU(3)$ gauge group and $16.5 > N_F > 8$ Dirac fermions transforming in the fundamental representation of $SU(3)$ [12], the number of thermal degrees of freedom is at least $g(T) = 8 \times 2 + 7/8 \times 4 \times 3 \times 8 = 100.0$, which is similar to the number of degrees of freedom in the SM, $g(T) = 106.75$. Therefore in thermal equilibrium, $\rho_{BZ} \approx \rho_{SM}$. Once T drops below the upper limit in Eq. (24), the SM and BZ sectors decouple. As the BZ fields lose energy via expansion they will evolve into continuous mass scalars, leaving an energy density in the stable scalar sector which is too large to dilute sufficiently by photon heating in the SM before nucleosynthesis. Therefore, a stable unparticle-based continuous mass scalar sector in general implies an upper limit on the temperature of the radiation-dominated Universe, which is of the order of 1 TeV for M_U close to its lower limit.

The above results are for the case $d_{BZ} = 3$. To show the sensitivity to d_{BZ} we calculate the lower bound on M_U from Eq. (10) for the cases $d_{BZ} = 2$ and 4:

$d_{BZ} = 2$

$$\begin{aligned} \mathbf{d}_U = \mathbf{1.1} : \quad M_U &\gtrsim 2600 \, k_1^{1/4} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{9/20} \text{ TeV} \\ \mathbf{d}_U = \mathbf{1.5} : \quad M_U &\gtrsim 1100 \, k_1^{1/4} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{1/4} \text{ TeV} \\ \mathbf{d}_U = \mathbf{2.0} : \quad M_U &\gtrsim 520 \, k_1^{1/4} \text{ TeV} . \end{aligned} \tag{25}$$

$d_{BZ} = 4$

$$\begin{aligned} \mathbf{d}_U = \mathbf{1.1} : \quad M_U &\gtrsim 43 \, k_1^{1/8} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{29/40} \text{ TeV} \\ \mathbf{d}_U = \mathbf{1.5} : \quad M_U &\gtrsim 33 \, k_1^{1/8} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{5/8} \text{ TeV} \\ \mathbf{d}_U = \mathbf{2.0} : \quad M_U &\gtrsim 23 \, k_1^{1/8} \left(\frac{\Lambda_U}{1 \text{ TeV}} \right)^{1/2} \text{ TeV} . \end{aligned} \tag{26}$$

Larger d_{BZ} for a given d_U reduces the lower bound on M_U for $\Lambda_U \approx 1 \text{ TeV}$, but increases the sensitivity to Λ_U . From Eq. (20), Eq. (25) and Eq. (26) we see that the lower bounds on M_U are in the range 20 TeV to 2600 TeV when $1.1 \leq d_U \leq 2.0$ and $2 \leq d_{BZ} \leq 4$, assuming that $\Lambda_U \gtrsim 1 \text{ TeV}$ and k_1 is not too different from 1.

3.4 Discussion

The first study of unparticle cosmology was presented in [8]. This was based on a purely dimensional estimate of the rate of production Γ_ψ of unparticles from SM radiation via a vector unparticle operator interaction of the form $\bar{\psi}\gamma_\mu\psi O_U^\mu$. It was assumed that once $\Gamma_\psi > H$, the unparticles and SM particles are in thermal equilibrium at T . Once the unparticles decouple, the unparticle sector is assumed to be stable and to have a temperature T_U which is less than the photon temperature T due to photon heating by subsequent annihilations. In order to ensure that T_U is sufficiently suppressed that the additional unparticle energy does not affect BBN, it was required that decoupling of the unparticles occurs at $T \gtrsim 1$ GeV, before the QCD phase transition. Requiring this imposes an upper bound on Λ_U . However, as we have shown, the actual suppression of the unparticle density relative to the radiation density is by at most a factor of 0.39, which is ineffective in suppressing a decoupled stable unparticle density with $\rho_U > 0.15\rho_{SM}$.

The main difference with our discussion of unparticle cosmology based on stable continuous mass scalars is in the interpretation of the condition $\Gamma_d > H$. Rather than establishing thermal equilibrium between the SM and the continuous mass scalar sector, we interpret this as a rapid flux of energy from SM radiation to the scalar sector, resulting in a scalar-dominated Universe. Therefore $\Gamma_d < H$ must hold for all T , imposing an upper bound on T during radiation-domination. Our analysis also differs in the interaction we considered, based on a scalar unparticle operator. The decay and annihilation rates in this case differ from a simple dimensional estimate due to the quark mass factors which must also be included.

Strong constraints on unparticles can be imposed by supernovae [5] and by unparticle-mediated long-range forces [6]. However, these constraints can be evaded by slightly by breaking the conformal invariance, for example by giving the deconstructing scalars a mass > 30 MeV to suppress their creation in supernovae [15]. This can be achieved by coupling the unparticle operators to Higgs scalars, in which case the continuous mass scalars have a lower bound m_{gap} on their masses but continuous masses at $m > m_{gap}$, which preserves their stability.

Since we have shown that $M_U \gtrsim 100$ TeV is typically necessary for compatibility with SM cosmology, it might appear that detection of unparticles in collider experiments is unlikely if continuous mass scalars serve as a good model for unparticle states. However, in the case where unparticle operators couple to the Higgs doublet, it is possible to detect unparticles with such large M_U e.g. by observing the partial Higgs boson decay width to gluon and photon pairs [16].

4 Conclusions

We have considered unparticle cosmology under the assumption that unparticle states can be modelled by continuous mass scalar particles. We have shown that the requirement that sufficiently stable (lifetime $\gtrsim 1$ s) continuous mass scalars do not dominate the energy density at nucleosynthesis implies that there is a lower bound on the scale M_U of the interaction between the Banks-Zaks fields and the Standard Model and an upper bound on the temperature of the radiation-dominated Universe.

For an interaction between SM quarks and scalar unparticle operators with $d_{BZ} = 3$ and $\Lambda_U \gtrsim 1$ TeV, the lower bound on M_U is typically of the order of 100 TeV for unparticle dimensions d_U in the range 1.1 - 2.0. Varying d_{BZ} widens the range of lower bounds, with $M_U \gtrsim 20 - 2600$ TeV for $2 \leq d_{BZ} \leq 4$. Decays will dominate over annihilations except if d_U is very close to 1. Including other unparticle operators can only strengthen these bounds.

A stable continuous mass scalar sector coupled to the Standard Model implies the existence of an upper bound on the temperature of the Standard Model radiation-dominated era. In the example with $d_{BZ} = 3$, the upper bound is of the order of 1 TeV for M_U close to its lower bound.

These constraints may be evaded if it is possible for the unparticle sector to come into thermal equilibrium initially and to subsequently decouple at $T \gtrsim M_W$. Then if the central charge of the unparticle CFT is sufficiently small, $c < 10$, the decoupled unparticle density will be diluted sufficiently to evade nucleosynthesis constraints. This requires a significant reduction in the number of effective degrees of freedom of

the unparticle CFT relative to the underlying Banks-Zaks sector, by approximately a factor of 10.

The significance of these conclusions for unparticle cosmology depends on the degree to which the unparticle states can be approximated by stable continuous mass particles. We have assumed that for the case of unparticles due to a Banks-Zaks sector with strong-coupling scale Λ_U , the unparticle states may be approximately described by continuous mass particles in the limit where Λ_U is large compared with the energy of interaction. However, this description cannot be exactly correct as CFTs do not have asymptotic states. A full CFT analysis of unparticle cosmology is ultimately necessary to establish the validity of the results suggested by the continuous mass scalar analysis.

In addition, we expect the scale-invariance of the strongly-coupled theory to be broken by the energy of unparticle interactions relative to Λ_U . This could then permit processes allowing the unparticle density to decay or annihilate to SM particles. Cosmological constraints are based on the unparticle density being stable until the time of nucleosynthesis. Therefore if the unparticle density has a lifetime greater than $O(1)$ s then the constraints derived in this paper will apply. A recent study [17] suggested that unparticles have a finite lifetime, whereas in [18] it was suggested that unparticle stability is possible, depending on the mass of the pole in the unparticle propagator relative to the mass of the possible SM final states in the decay process. It will be important to establish if unparticles can be sufficiently unstable to evade cosmological constraints from nucleosynthesis.

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